

A linearly graded pn junction in the depletion approximation

In a linearly graded pn junction, the doping varies linearly $N_D(x) - N_A(x) = \alpha x$. In the depletion approximation, it is assumed that there is a depletion width W around the transition from p to n where the charge carrier densities are negligible. Outside the depletion width the charge carrier densities are equal to the doping densities so that the semiconductor is electrically neutral outside the depletion width. Using these approximations, the charge density in the depletion region is,

$$\rho = e\alpha x \quad \text{for} \quad -\frac{W}{2} < x < \frac{W}{2}, \quad \rho = 0 \quad \text{otherwise.}$$

The charge density can be integrated to determine the electric field $E = \int \frac{\rho}{\epsilon} dx$,

$$E(x) = -\frac{e\alpha}{2\epsilon} \left(\left(\frac{W}{2} \right)^2 - x^2 \right).$$

The electric field can be integrated to determine the electrostatic potential $\phi = -\int E dx$,

$$\phi(x) = -\frac{e\alpha}{2\epsilon} \left(\left(\frac{W}{2} \right)^2 x - \frac{x^3}{3} \right).$$

The voltage across the diode is the difference of the electrostatic potential across the depletion width,

$$V_{bi} - V = \frac{e\alpha W^3}{12\epsilon}.$$

The built-in voltage is related to the doping concentrations at the edge of the depletion zone, $N_{W/2} = N_A(-W/2) = N_D(W/2)$, by the usual formula,

$$V_{bi} = k_B T \ln \left(\frac{N_{W/2}^2}{n_i^2} \right).$$

The depletion width and the gradient in the doping can then be calculated,

$$W = \sqrt{\frac{6\epsilon(V_{bi} - V)}{eN_{W/2}}} \quad \text{and} \quad \alpha = \frac{2N_{W/2}}{W}.$$

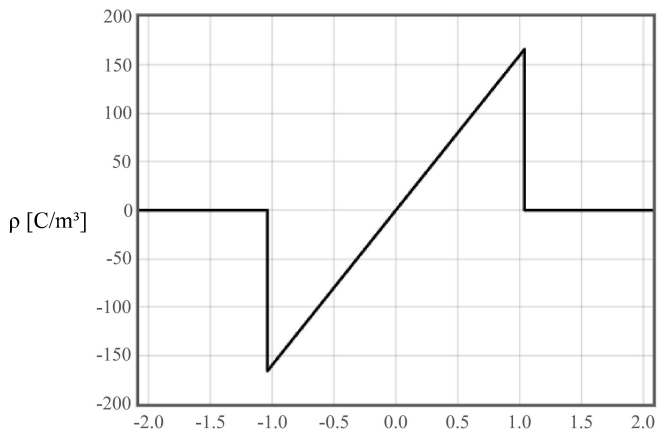
The form below determines W and $N_{W/2}$ numerically by guessing a value for $N_{W/2}$ and calculating the corresponding α until the correct value is guessed.

$N_v(300) = $	<input type="text" value="9.84E18"/>	cm^{-3}	$\alpha = $	<input type="text" value="1E19"/>	cm^{-4}	$E_g = $	<input type="text" value="1.166-4.73E-4*T*(T+636)"/>	eV			
$N_c(300) = $	<input type="text" value="2.78E19"/>	cm^{-3}	$N_{W/2} = $	<input type="text" value="1.04e+15"/>	cm^{-3}	$\epsilon_r = $	<input type="text" value="12"/>	$T = $	<input type="text" value="300"/>	K	
$\mu_p = $	<input type="text" value="480"/>	$\text{cm}^2/\text{V s}$	$\mu_n = $	<input type="text" value="1350"/>	$\text{cm}^2/\text{V s}$	$\tau_p = $	<input type="text" value="1E-10"/>	s	$\tau_n = $	<input type="text" value="1E-10"/>	s
	$V = $	<input type="text" value="-0.5"/>	V							<input type="button" value="Submit"/>	

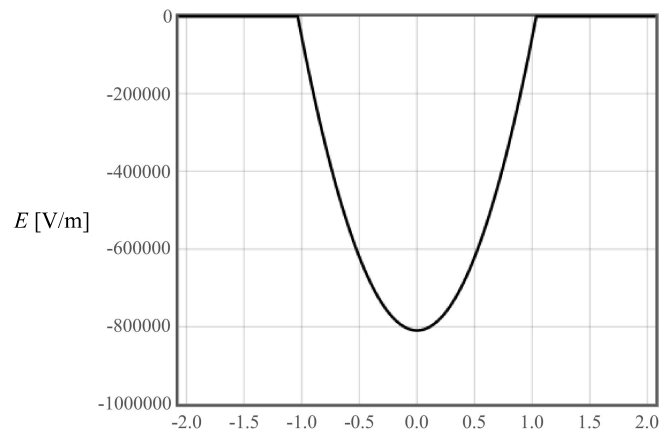
$$E_g = 1.12 \text{ eV} \quad W = 2.07 \text{ } \mu\text{m} \quad \alpha = 1.00\text{e}+19 \text{ cm}^{-4} \quad N_{W/2} = 1.04\text{e}+15 \text{ cm}^{-3} \quad V_{bi} = 0.620 \text{ V} \quad C_j = 5.12 \text{ nF/cm}^2$$

$$D_p = 12.4 \text{ cm}^2/\text{s} \quad D_n = 34.9 \text{ cm}^2/\text{s} \quad L_p = 0.352 \text{ } \mu\text{m} \quad L_n = 0.591 \text{ } \mu\text{m}$$

Charge density

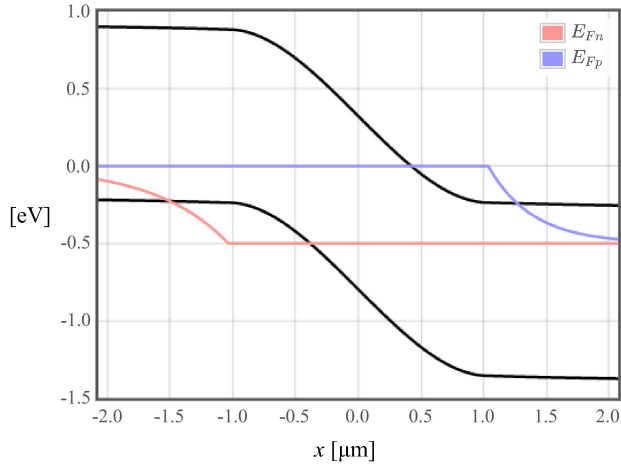


Electric field



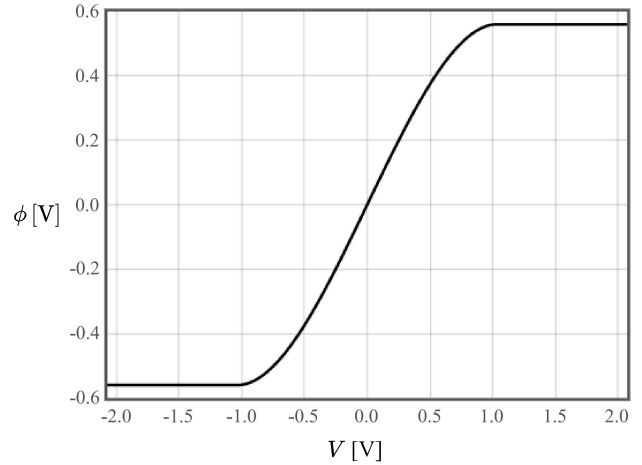
x [μm]

Band diagram

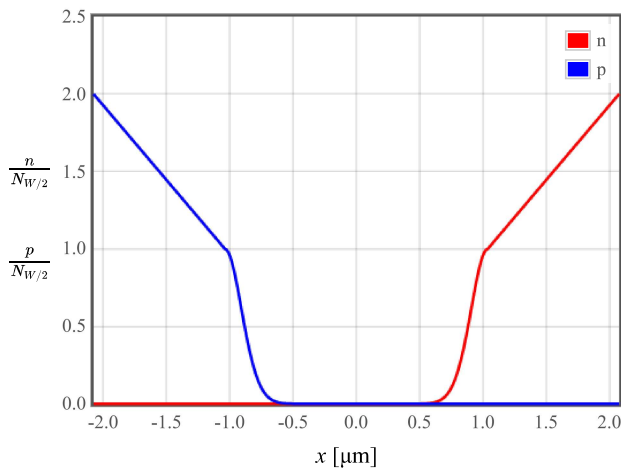


x [μm]

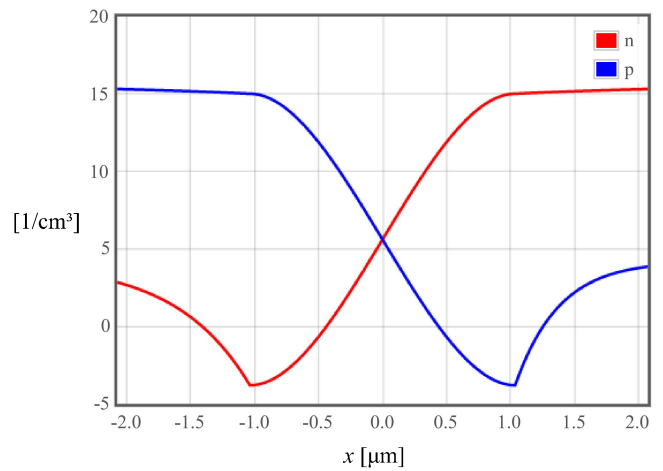
Electrostatic potential



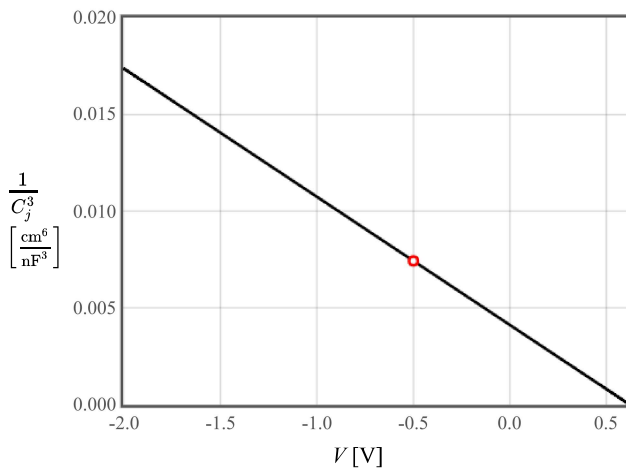
Carrier Densities



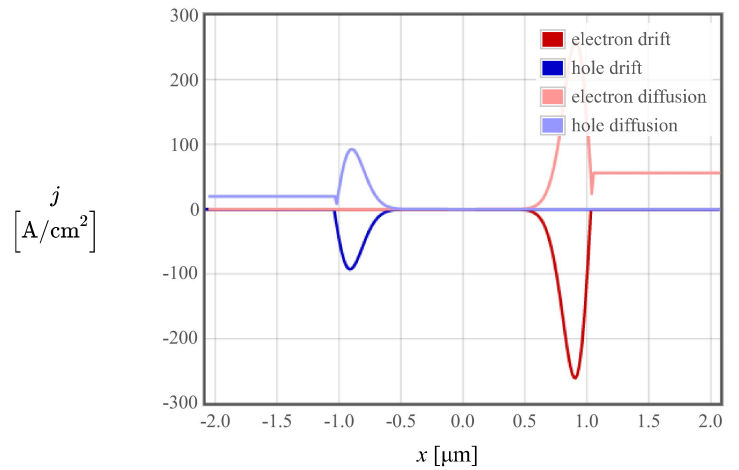
log(Carrier Densities)



Capacitance - Voltage



Current densities



Combining the equations $V_{bi} - V = \frac{e\alpha W^3}{12\epsilon}$ and $C_j = \frac{\epsilon}{W}$,
it can be shown that $\frac{1}{C_j^3} = \frac{12(V_{bi} - V)}{\epsilon^2 e \alpha}$.

$$\vec{j}_{n,\text{drift}} = ne\mu_n\vec{E}, \quad \vec{j}_{p,\text{drift}} = pe\mu_p\vec{E},$$

$$\vec{j}_{n,\text{diffusion}} = eD_n\frac{dn}{dx}, \quad \vec{j}_{p,\text{diffusion}} = -eD_p\frac{dp}{dx}$$